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The Study of Navier Slip Condition on the Flow and Heat Transfer in a Coolant Surrounded an Exponentially Stretching Sheet

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Abstract

The paper presents the study of velocity profiles in a hydrodynamic flow and heat transfer in a Newtonian fluid over an exponentially stretching sheet. Navier slip condition is used at the boundary. The stretching of the sheet is assumed to be nonlinearly proportional to the distance from slit. Non-linear partial differential equations characterize the flow phenomenon with boundary conditions in a semi infinite domain. The equations are transformed to nonlinear ordinary differential equations by applying suitable local similarity transformation. The series solution of the transformed equations are obtained by using differential transform method and Pade approximation with assistance from the shooting method in obtaining the unknown initial values. The solution is obtained in a power series with assured convergence. The effects of various parameters on velocity and temperature profiles are presented graphically.

Keywords: MHD, Exponential stretching sheet, Newtonian fluid, Navier slip, Differential transform method.

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1.Introduction

The study of boundary layer behaviour over a stretching sheet occurring in several engineering applications and manufacturing processes of thin films in computer industry, in space applications etc. The practical applications of continuous flat surfaces are in aerodynamic extrusion of plastic sheets, rolling and manufacturing of plastic films, cooling of metallic plates and boundary layer flow over heat treated materials between feed roll and a windup roll. Sakiadis [1] initiated the study of boundary layer over a continuous solid surface moving with constant speed. Crane [2] studied the two dimensional boundary layer flow due to a stretching sheet. He assumed the velocity of the sheet to vary linearly with the axial distance. After this pioneering work, the flow over a stretching surface has drawn considerable attention and a good amount of literature in different field has been generated on this problem. The common feature in all these studies is that the flow field obeys the no-slip condition at the boundary. But in certain situations the no-slip condition is required to be replaced by the Navier slip boundary condition.

Anderson [3] studied the effects of slip boundary condition on the flow of Newtonian fluid past a stretching sheet. Bidin *et al* [4] analyzed the boundary layer flow over a stretching sheet with a convective boundary condition and slip effect, Chethan, *et al* [5] studied the flow and heat transfer of an exponential stretching sheet in a viscoelastic liquid with navier slip boundary condition, Fang *et al* [6] obtained the exact solution of MHD flow under the slip condition over a permeable stretching sheet, Fang, T. J *et al* [7] Slip MHD viscous flow over a stretching sheet – an exact solution, Sahoo *et al* [8] studied Flow and heat transfer of a third grade fluid past an exponentially stretching sheet with partial slip boundary conditions, ,Sajid *et al* [9] studied stretching flows with general slip boundary condition ,Wang[10] analyzed the flow due to a stretching boundary with partial slip-an exact solution of the Navier-Stokes equations and Analysis of viscous flow due to a stretching sheet with surface slip .

We have chosen to study navier slip condition on the flow and heat transfer in a coolant surrounded by an exponentially stretching sheet. The Differential transform method along with Padė approximation is used to obtain a convergent series solution.

Nomenclature

c_p	specific heat at constant pressure	**Greek symbols**	
E	eckert number	η	similarity variable
k	thermal conductivity	υ	kinematic viscosity
l	reference length	μ	dynamic viscosity
T	fluid temperature of the moving sheet	θ	dimensionless temperature in PEST case
T_w	wall temperature	ϕ	dimensionless temperature in PEHF case
T_∞	temperature far away from the sheet	f	dimensionless stream function
U_0	constant	σ	electrical conductivity
U_w	stretching velocity of the boundary	ρ	density of the fluid
u,v	velocity components along x and y directions	η	similarity variable
x	flow directional co ordinate along stretching sheet .	**Subscripts**	
y	distance normal to the stretching sheet	w	wall temperature
X,Y	dimensionless coordinates	∞	ambient temperature conduction

2. Mathematical formulation

The governing equations and the boundary conditions for momentum and heat transfer of the stretching sheet problem are:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{2.1}$$

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial x}=\upsilon\frac{\partial^2 u}{\partial x^2}-\left(\frac{{\mu_m}^2\sigma {H_0}^2}{\rho}\right) \tag{2.2}$$

$$\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\frac{\kappa}{\rho c_p}\left(\frac{\partial^2 T}{\partial y^2}\right)+\frac{\kappa}{\rho c_p}\left(\frac{\partial u}{\partial y}\right)^2 \tag{2.3}$$

where u and v are the velocity components of the fluid in x and y directions respectively, υ is the kinematic coefficient of viscosity, μ_m is the magnetic permeability, σ is the electrical conductivity, H_0 is the applied magnetic field, ρ is density of the fluid, T is the temperature of the fluid, k is the thermal conductivity of the fluid and c_p is the specific heat at constant pressure.

The flow is generated solely by stretching the boundary surface in the x direction, we employ the following boundary conditions with the stretching assumed to be in exponential proportion to the axial coordinate. Following Elbashbeshy [6], we employ the following boundary conditions on velocity and temperature are

$$u=U_w(x)=U_0 exp\left(\frac{x}{l}\right)+\chi\upsilon\frac{\partial u}{\partial y} \quad , v=0 \qquad at \;\; y=0$$

$$\left.\begin{cases} T=T_w=T_\infty+\left(T_w-T_\infty\right)e^{\frac{x}{l}} & in \;\; PEST \\ \dfrac{\partial T}{\partial y}=-\dfrac{Dl}{k\sqrt{Re}}X^2 & in \;\; PEHF \end{cases}\right\} y=0$$

$$u\to 0, \;\; T\to T_\infty \;, \;\; y\to\infty \tag{2.4}$$

Introducing the stream function $\psi(x,y)$ defined by :

$$U=\frac{\partial\psi}{\partial Y}, \quad V=-\frac{\partial\psi}{\partial X} \tag{2.5}$$

into the equations (2.2) and (2.3) we get

$$-\gamma\frac{\partial^3\psi}{\partial Y^3}+\frac{\partial\psi}{\partial Y}\frac{\partial^2\psi}{\partial X\partial Y}-\frac{\partial\psi}{\partial X}\frac{\partial^2\psi}{\partial Y^2}+\left(\frac{\mu_m{}^2\sigma H_0{}^2}{\rho}\right)\frac{\partial\psi}{\partial Y}=0 \tag{2.6}$$

$$\frac{\partial\psi}{\partial Y}\frac{\partial\overline{T}}{\partial X}-\frac{\partial\psi}{\partial X}\frac{\partial\overline{T}}{\partial Y}=\frac{1}{pr}\frac{\partial^2\overline{T}}{\partial Y^2}+\left(\frac{\partial^2\psi}{\partial Y^2}\right)^2 \tag{2.7}$$

The corresponding boundary conditions becomes

$$U=U_w(x)=U_0e^X+\chi\upsilon\frac{\partial^2\psi}{\partial Y^2}\ ,\quad V=0\quad at\quad Y=0$$

$$\left.\begin{cases} T=T_w=T_\infty+(T_w-T_\infty)e^X & in\quad PEST \\ \dfrac{\partial T}{\partial Y}=-\dfrac{Dl}{k\sqrt{Re}}\,X^2 & in\quad PEHF \end{cases}\right\}\quad at\quad Y=0$$

$$\frac{\partial\psi}{\partial Y}\to 0,\quad T\to T_\infty\quad as\quad Y\to\infty$$

(2.8)

Solution of Momentum equation:

The following transformation is used to convert the partial differential equation into an ordinary differential equation

$$\psi(X,Y)=\sqrt{2Re}\ f\left[Y\sqrt{\frac{Re}{2}}\,exp\left(\frac{X}{2}\right)\right]exp\left(\frac{X}{2}\right) \tag{2.9}$$

where $\eta=Y\sqrt{\frac{Re}{2}}\,exp\left(\frac{X}{2}\right)$ is the similarity variable , $Re=\frac{U_0 l}{\upsilon}$ is the Reynolds number. Substituting in (2.7), we obtain a nonlinear boundary value problem

$$f_{\eta\eta\eta}-2f_\eta{}^2-f\ f_{\eta\eta}-2Q\ f_\eta=0 \tag{2.10}$$

The boundary equations becomes

$$f=0,\ \ f_{\eta}=1+k\,f_{\eta\eta}\quad at\quad \eta=0$$
$$f_{\eta}\to 0\qquad as\qquad \eta\to\infty \qquad (2.11)$$

we have assumed the following condition to solve third order dofferentail equation

$$f_{\eta\eta}=\alpha\quad at\quad \eta=0$$

Method of Solution

We adopt the shooting method with Runge-Kutta- Fehlberg 45 scheme to solve the initial value problems in PEST and PEHF. The coupled non-linear equations (2.12) - (2.15) are transformed to a system of first order ordinary differential equations using $f=F_1$ in the following form :

$$\frac{dF_1}{d\eta}=F_2 \qquad F_1(0)=0,$$
$$\frac{dF_2}{d\eta}=F_3 \qquad F_2(0)=1+k\,F_3(0),$$
$$\frac{d\,F_3}{d\eta}=2\,F_2{}^2-F_1\,F_3+2\,Q\,F_2 \qquad F_3(0)=\alpha.$$

(2.12)

The above boundary value problem is converted to an initial value problem by choosing the value of $F_3(0)$ and appropriately. Resulting initial value problem is integrated using RK Felberg 45 method. The constant guess which satisfies the boundary condition for different slip factors is

	α		
Q	K = 0	K= 0.01	K = 0.05
1	-1.912647983	-1.8689930529	-1.714468910
2	-2.379420850	-2.3163599348	-2.096951788
3	-2.768210449	-2.6859864501	-2.403595029
4	-3.110866887	-3.0073557281	-2.663608408

The differential transform of $f(z)$

$$F[k] = D^k[f(z)] = \frac{1}{k!}\left[\frac{d^k}{dz^k}f(z)\right]_{z=z_0}$$

where is *f(z)* the original function and F[k]is the transformed function.

Applying differential transform to (2.10) and (2.11), we get a recurrence relation as

$$(k+1)(k+2)(k+3)F(k+3) + 2(k+1)F[k+1](k+1-r)F[k+1-r] + \sum_{r=0}^{k} F[r](k+1-r)(k+2-r)F(k+2-r) + 2Q(k+1)F[k+1] = 0 \quad (2.13)$$

The corresponding boundary conditions becomes

$$F[0] = 0, \;\; F[1] = 1 + k\alpha$$

By using inverse differential operator, we get

$$f(\eta) = F[0] + F[1]\eta + F[2]\eta^2 + F[3]\eta^3 + \dots\dots\dots\dots\dots \quad (2.14)$$

where

$$F[2] = \frac{\alpha}{2},$$

$$F[3] = \frac{1}{6}\left[2Q(1+k\alpha) + 2(1+k\alpha)^2\right],$$

$$F[4] = \frac{1}{24}\left[2Q\alpha + 3\alpha(1+k\alpha)\right]$$ and so on, are calculated using Mathematica.

Differentiating w.r.t to η, we get to

$$f'(\eta) = F[1] + 2F[2]\eta + 3F[3]\eta^3 + \dots\dots\dots\dots\dots \quad (2.15)$$

As the radius of convergences of the obtained power series is not large enough to contain both boundaries, to obtain the same , we need to make use of Pade approximation.

- Pade approximation of order [5,7] has been used for the slip factor k = 0

- Pade approximation of order [6,7] has been used for the slip factor k = 0.01
- Pade approximation of order [5,6] has been used for the slip factor k = 0.05

3. Heat transfer analysis

We consider two general cases of non-isothermal boundary conditions, namely

- Prescribed exponential order surface temperature (PEST)
- Prescribed exponential order heat flux (PEHF)

Prescribed exponential order surface temperature (PEST)

We define a non dimensional temperature $\theta(\eta)$ as

$$\theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty} \tag{3.1}$$

where $T_w-T_\infty=e^X$ and $T-T_\infty=\theta(\eta)\,e^X$

using (3.1) in the (2.7) and (2.8), we obtain a nonlinear ordinary differential equation for

$$\theta_{\eta\eta}+Pr\ f\ \theta_\eta-2\ Pr\ f_\eta\ \theta+E\ Pr\ f_{\eta\eta}{}^2=0 \tag{3.2}$$

$$\theta=1 \quad at \quad \eta=0$$
$$\theta\to 0 \quad as \quad \eta\to\infty \tag{3.3}$$

We have assumed the following condition to solve the second order differential equation

$$\theta_\eta=\beta \quad at \quad \eta\to 0$$

Applying DTM (3.2) and (3.3) we get

$$(k+1)(k+2)F(k+2)+\ pr\ \sum_{r=0}^{k}(r+1)G[r+1]\ F[k-r]+2\ pr\ \sum_{r=0}^{k}(r+1)F(r+1)G[k-r]+$$
$$pr\ E\ \sum_{r=0}^{k}(r+1)(r+2)F[r+2]\ (k+1-r)(k+2-r)F(k+2-r)=0 \quad (3.4)$$

$$G[0]=1\ ,G[1]=\beta$$

By using inverse differential operator, we get

$$\theta(\eta) = G[0] + G[1]\,\eta + G[2]\,\eta^2 + G[3]\,\eta^3 + \ldots\ldots\ldots\ldots\ldots \tag{3.5)}$$

using Mathematica, we get

$$G[2] = \frac{1}{2}\left(-0.5\,\alpha^2 + 2(1 + k\,\alpha)\beta\right)$$

$$G[3] = \frac{1}{6}\left(1 + k\alpha - 1.0\,\alpha\left(2Q(1 + k\alpha) + 2(1 + E\,k\,\alpha)^2 + 2(-1 - k\,\alpha\,\beta)\right)\right)$$

and so on

To get the convergence of the power series obtained by DTM, Pade approximation is used.

Prescribed exponential order heat flux (PEHF)

We define a non dimensional temperature $\phi(\eta)$ as $\phi(\eta) = \dfrac{T - T_\infty}{T_w - T_\infty}$ (3.6)

$$T - T_\infty = \frac{T}{k}\sqrt{\frac{2}{Re}} \quad , \quad T_w - T_\infty = \frac{T_1 l}{k}\sqrt{\frac{2}{Re}}\,e^{\frac{3x}{2}} \tag{3.7}$$

Using (3.6), and (3.7) in (2.7),we obtain a second order nonlinear differential equation for $\phi(\eta)$ as

$$\varphi_{\eta\eta} + Pr\ f\ \phi_\eta - 2\ Pr\ f_\eta\ \phi + \ Pr\ E_s\, f_{\eta\eta}{}^2 = 0 \tag{3.8}$$

The boundary conditions are

$$\phi_\eta = -1 \quad at \quad \eta = 0$$
$$\phi \to 0 \quad as \quad \eta \to \infty \tag{3.9}$$

We have assumed the following condition to solve the second order differential equation

$$\theta = \gamma \quad at \quad \eta = 0$$

Applying DTM (3.8)and (3.9) we get

$$(k+1)(k+2)F(k+2)+\ pr\ \sum_{r=0}^{k}(r+1)H[r+1]\ F[k-r]+2\ pr\ \sum_{r=0}^{k}(r+1)F(r+1)H[k-r]+$$

$$pr\ E_s\ \sum_{r=0}^{k}(r+1)(r+2)F[r+2]\ (k+1-r)(k+2-r)F(k+2-r)=0 \qquad (3.10)$$

$$H[0]=\gamma,\ H[1]=-1$$

By using inverse differential operator, we get

$$\phi\ (\eta)=H\ [0]+H\ \ [1]\ \eta+H[2]\ \eta^2\ +H[3]\ \eta^3\ +................ \qquad (3.11)$$

using Mathematica, we get

$$H[2]=\frac{1}{2}\left(-0.5\alpha^2+2(1+k\,\alpha)\gamma\right)$$

$$H[3]=\frac{1}{6}\left(1+k\alpha-1.0\,\alpha\left(2Q(1+k\alpha)+2(1+E\,k\,\alpha)^2+2(-1-k\,\alpha\,\gamma)\right)\right)$$

To get the convergence of the power series obtained by DTM, pade approximation is used.

K=0.01			
Q	α	**PEST** β	**PEHF** γ
1	-1.8689930529	-0.722183611	0.2675571680
2	-2.3163599348	-0.500314385	1.5521492451
3	-2.6859864501	-0.340471233	1.8113148012
4	-3.0073557281	-0.209716105	1.9734189281

As the radius of convergences of the obtained power series is not large enough to contain both boundaries, to obtain the same, we need to make use of Pade approximation.

- Pade approximation of order [6/7] has been used for the slip factor k = 0.01
- Pade approximation of order [5/6] has been used for the slip factor k = 0.05

Results and discussions

The navier slip condition at the boundary on the flow and heat transfer in a coolant surrounded an exponentially stretching sheet is analysed. In figures , the graphs of $f(\eta)$ and $f'(\eta)$, $\theta(\eta)$ and $\varphi(\eta)$ versus η are drawn for different values of the parameters $Q, E(E_s), Pr$ in both PEST and PEHF cases.

- An increase in Q is to reduce the velocity in the boundary layer which results in thinning of the boundary layer thickness and increasing the thermal boundary layer thickness.
- The increase in $E(Es)$ is to enhance the temperature .This is due to the fact that the heat energy is stored in the liquid considered due to frictional heating.
- An increase in Pr is associated with a decrease in the temperature. Thermal boundary layer thickness decreases with increase in the values of Pr. The increase of Prandtl number means there is a slow rate of thermal diffusion. The temperature is at unity on the will in PEST case where as it may be other than unity in PEHF case.

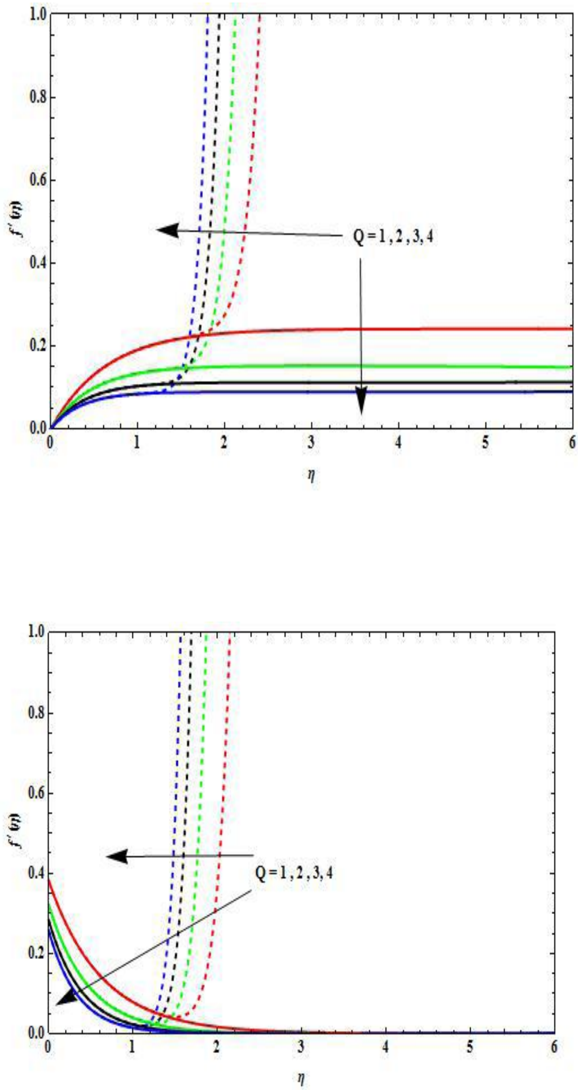


Fig 1: Plots of $f(\eta)$ and $f'(\eta)$ for different values of Q for k = 0.01

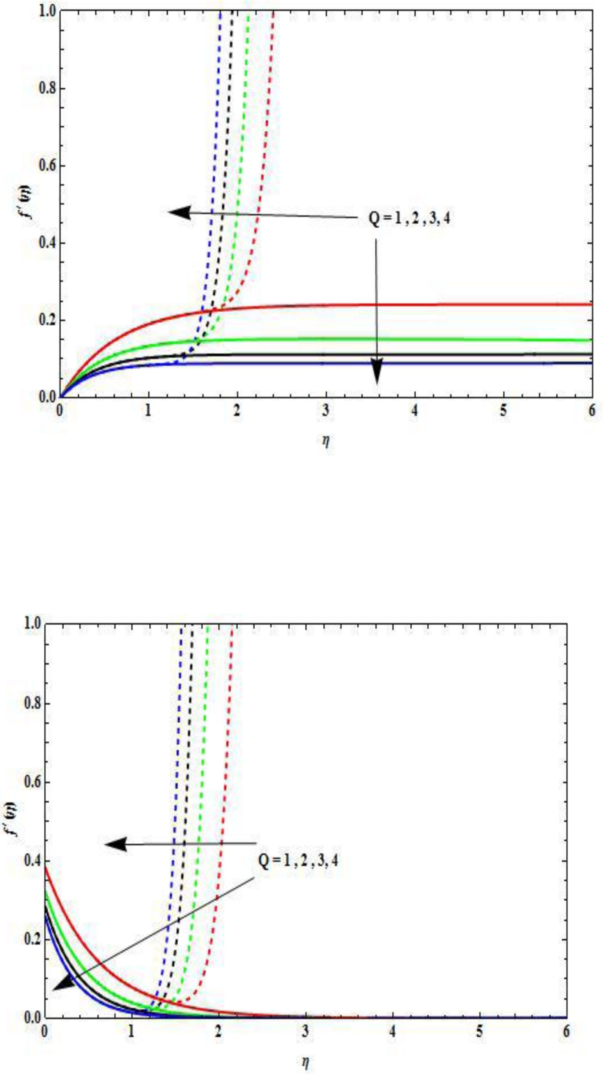
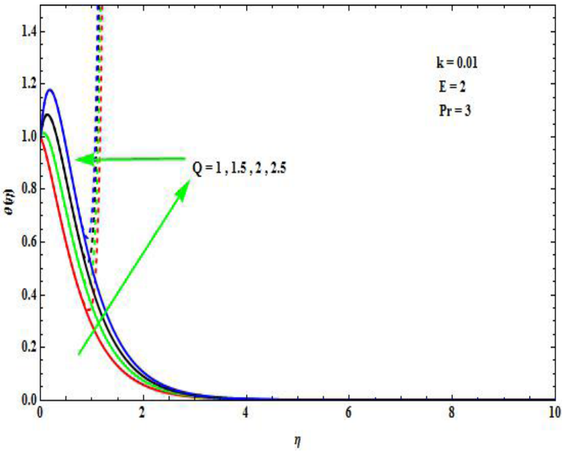


Fig 2 : Plots of $f(\eta)$ and $f'(\eta)$ for different values of Q for k = 0.05



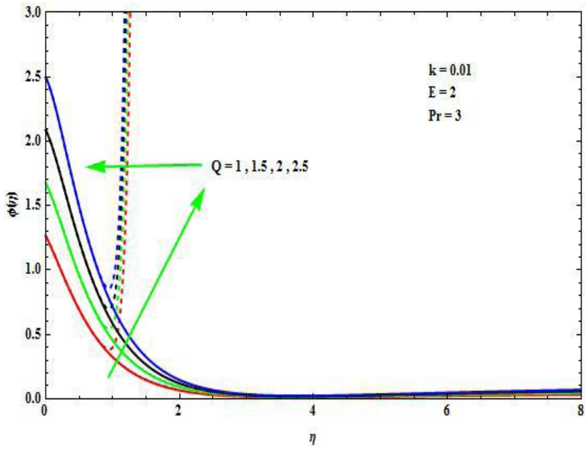
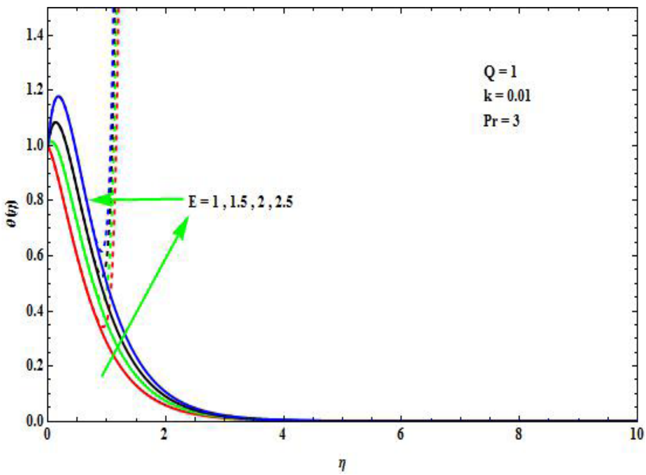


Fig 3: Plots of $\theta(\eta)$ and $\phi(\eta)$ for different values of Q in PEST and PEHF cases.



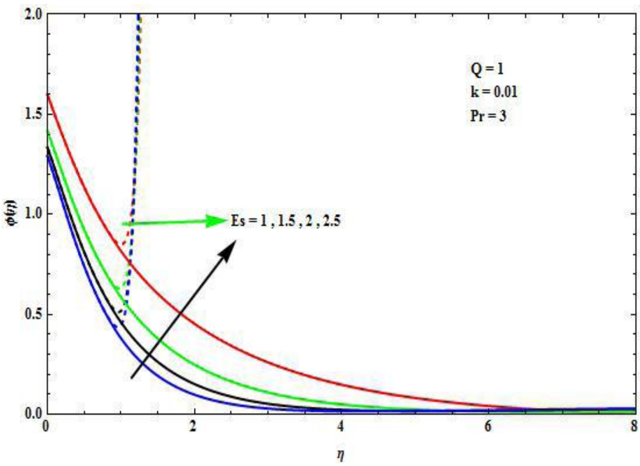
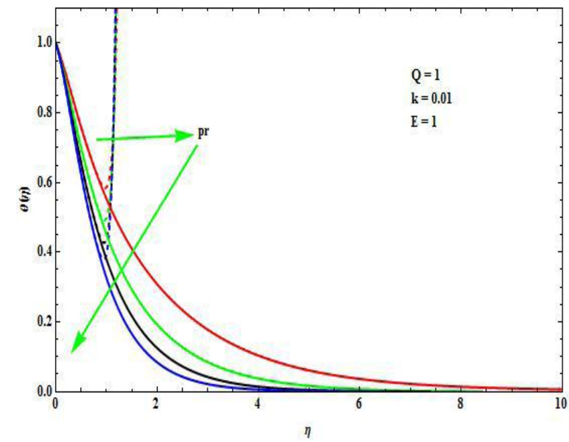


Fig 4: Plots of $\theta(\eta)$ and $\phi(\eta)$ for different values $E\left(E_s\right)$of in PEST and PEHF cases.



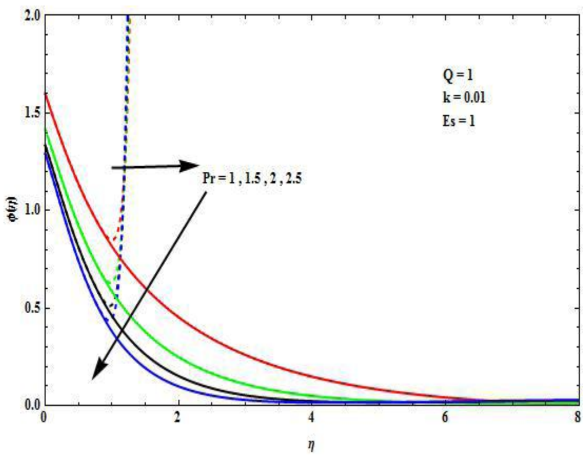


Fig 5 : Plots of $\theta(\eta)$ and $\phi(\eta)$ for different values Pr of in PEST and PEHF cases.

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